STAT2001 midterm exam solution for 2019-20 T1

1. (a) Suppose that in a hospital, number of babies born follows a Poisson process with a rate of 5 births per day. What is the probability that at most 2 babies are born during the next 2 hours?

(b) Bowl I contains 7 red chips and 3 blue chips. 5 of these 10 chips are chosen at random and without replacement and put in bowl II, which was originally empty. 1 chip is then chosen at random from bowl II. Given that this chip is blue, find the conditional probability that 2 red chips and 3 blue chips are transferred from bowl I to bowl II.

Answer

(a) Let X be the number of babies born in the next 2 hours. X~Poisson(5/12). So

(b) Let A denote {0 blue from I to II}, B denote {1 blue from I to II}. C denote {2 blue from I to II}, D denote {3 blue from I to II}, F denote {Blue from II}

1. (a) Three distinct integers are chosen at random from the first 20 positive integers (i.e. 1,2,3,…,20). Compute the probability that the product of the chosen integers is even.

(b) Let *C1, C2* and *C3* be mutually independent events with probabilities and respectively. Compute .

Answer:

(a)

(b)

1. Let *X* be a discrete random variable having the following probability mass function.
2. Find the moment generating function of *X*.
3. Find the expected value of *X*.
4. Find the variance of *X*.

Answer:

Remark:

1, For (b) and (c), of course you can differentiate on M(t) to get the same results.

2, If you can notice that X=Y+1 where Y is Poisson(3), you can finish the whole question very quickly.

1. A continuous random variable *X* has the following probability density function.
2. Find the mean of *X.*
3. Find the variance of *X*.
4. Let µ and σ be the mean and standard deviation of *X* respectively.

Find *P(µ-3σ<X< µ+3σ)*.

1. Find the cumulative distribution function of *X*.

Answer:

1. *µ-3σ=-0.1708, µ+3σ=1.1708. P(-0.1708<X<1.1708)=1.*
2. Let *X* be a continuous random variable following Exponential distribution with parameter Find .

Answer: